Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

Let the circle be tangent at the point $P\equiv(a\cos\theta,\ b\sin\theta)$ to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

and let the chord of contact of the other two points touch at Q the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\lambda^2.$$

Since the tangents at P and Q are equally inclined to the axes, the coördinates of Q are $(\lambda a\cos\theta,\ -\lambda b\sin\theta)$, or $(-\lambda a\cos\theta,\ \lambda b\sin\theta)$. Since a chord of a conic, tangent to a similar coaxial conic, is bisected at the point of contact, the center of the circle lies on the normal at Q; and also lies on the normal at P; *i. e.*, on

$$\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=\pm\lambda(a^2-b^2),\text{ and }\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2.$$

Hence the required locus is

$$\frac{4a^2x^2}{(1\pm\lambda)^2}+\frac{4b^2y^2}{(1\mp\lambda)^2}=a^2-b^2.$$

PROBLEMS FOR SOLUTION.

ALGEBRA.

270. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics in The University of Pennsylvania, Philadelphia, Pa.

Find the simplest integral form of the sum $y(y-1)\ldots(y-x)+2y(2y-1)\ldots(2y-x)+\ldots+zy(zy-1)\ldots(zy-x)$.*

271. Proposed by L. E. NEWCOMB, Los Gatos, California.

Sum the series $\dfrac{a}{b}+\dfrac{a^3}{3b^3}+\dfrac{a^5}{5b^5}+\ldots$ to ∞, $b>a$.

272. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that the relations $x=\dfrac{ar+bs}{\lambda}=\dfrac{as-br}{\mu}=\dfrac{a\gamma-b\mu}{r}=\dfrac{a\mu+b\lambda}{s}$ between the finite real quantities x, a, b, r, s, λ, μ requires that $x^2=a^2+b^2$.

*This series is of frequent occurrence in certain investigations in Group Theory. Ed.